UNL Putnam Exam Study Seminar

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In the Putnam Exam, you are given miscellaneous problems. To solve these problems, in my opinion, you need 2 things:

- Proof methods: Proof by Contradiction, Induction, Pigeonhole Principle, Invariants...
- Background knowledge:
  - Basic: Algebra, Calculus...
  - Picked up from upper-level math courses: Combinatorics, Analysis, Linear Algebra, Number Theory...

A few problems require basic knowledge only. Some problems will need upper-level math knowledge.

The more practice and the more math you learn, the better you get!

Today, we will investigate **Proof by Contradiction**!

Problem 1. Prove that $\sqrt{2}+\sqrt{3}+\sqrt{5}$ is an irrational number.

Problem 2. Show that no set of nine consecutive positive integers can be partitioned into two sets with the product of the elements of the first set equal to the product of the elements of the second set.

Problem 3. Every point of the three-dimensional space is colored red, green, or blue. Prove that one of the colors attains all distances, meaning that any positive real numbers represents the distance between some two points of this color.

Problem 4. The union of nine planar surfaces, each of area equal to 1, has a total area equal to 5. Prove that the overlap of some two of these surfaces has an area greater than or equal to $\frac{1}{9}$.