# UNL Putnam Exam Study Seminar

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Today, we will do some problems involving **Extremal Heuristic**.

Sometimes, the key to solve a math problem lies in the extremal (maximal or minimal) elements of a set. Therefore, looking at extremal elements is a useful heuristic in your problem-solving toolbox.

Some other heuristics: searching for a pattern, dividing into cases, arguing by contradiction...

Problem 1 (Warmup). Imagine an infinite chessboard that contains a positive integer in each square. If the value in each square is equal to the average of its four neighbors to the north, south, west, and east, prove that the values in all the squares are equal.

Problem 2. Complete the following square with integers between 1 and 9 such that the sum of the numbers in each row, column, and diagonal is as indicated.
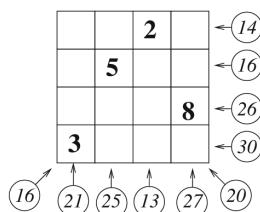
Problem 3. Prove that it is impossible to dissect a cube into finitely many cubes, no two of which have the same size.

Problem 4. In the plane we have marked a set S of points with integer coordinates. We are also given a finite set V of vectors with integer coordinates. Assume that S has the property that for every marked point P, if we place all vectors from V with origin at P, then more of their ends are marked than unmarked. Show that the set of marked points is infinite.